CS205 – Class 6

Reading: Heath 3.6 (p137-143), 4.7 (p202)

Numeric Linear Algebra Summary

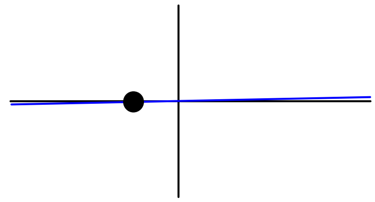
When your matrix A is:

1. Non-singular use LU decomposition.
2. Over determined use QR with Householder.
3. Under determined use SVD. This means some of your variables don't have any meaning on your solution, i.e. where you parked your car.
4. Principal Component Analysis
    a. PCA let's you throw away 10,000 terms and keep 6.
    b. But **don't** use the SVD (too slow and gives you everything)! Instead, find your singular values (i.e. σ_i) using the power method. Then can get eigenvectors of A^TA and AA^T using division.
5. For a linear system Ax=b, if the column space of A doesn't span b, there will be no exact solution but a least square solution. But by adding variables into the system, and thus adding corresponding columns into A, such that the augmented column set happens to span b, the system will eventually get a unique solution.

Covered in class*:* 1, 4, 5, 7, 8
Reading*:* Heath 5.1-5.5

Nonlinear Equations

1. Nonlinear equations are much more difficult to solve than simple linear equations, thus we will move from systems of equations to scalar equations to get started.
    a. Before we move from linear to nonlinear equations, let's first move from systems of equations to scalar equations in the linear case to get warmed up.
        i. consider ax=b where a and b are merely numbers
            1. of course the solution to this is simply x=b/a
            2. the only worry here would be if a was small or "zero" in which case the det(A)=det(a) is zero and the matrix [a] is essentially singular or near singular
        ii. let's take a different, functional look at this
            1. assume that we had a straight line y=ax+b and that we wanted to find the roots where y=0, then we need to solve 0=ax+b or ax=-b
            2. This is our linear equation with solution x=-b/m and now we see that a small slope is equivalent to ill-conditioning
                a. Consider y=(1e-16)x-1e-16 where the root is x=1 but the division x=(1e-16)/(1e-16) looks like 0/0 and is indeterminate and full of errors.



b.

b. Now for nonlinear equations we are solving A(x)=b where A is a vector valued function of x.
   i. Moreover we can write A(x)=0 since the b term can be incorporated into the vector valued function A of x.
   ii. As a scalar equation, we have a(x)=0 which looks odd, so we switch notation to f(x)=0.
   iii. Note how this looks like a root finding problem. In general, any g(x)=b can be rewritten as f(x)=0 by moving b to the left hand side.
      1. There may be any number of roots from 0,1,2, etc. to infinity.
      2. Remember “roots”=”solutions” from here on out.
   iv. Since $f'(x^*)$ is the slope of the function f at and “near” a root x^*, $f'(x^*)$ gives us an indication of the condition number. That is, it takes the place of “a” in the linear case.
      1. If $f'(x^*)$ is small or “zero” then we could be in trouble. That is, the condition number is large. Moreover the function is locally *flat.*
      2. A simple root has $f'(x^*) \neq 0$.
      3. A multiple root has $f'(x^*) = 0$ which could be problematic.

2. Previously when looking at systems of linear equations, we introduced direct methods like Gaussian Elimination and the Cholesky factorization. The other kind of method is called an iterative method where one starts with an initial guess x_1 and iterates through a sequence x_1, x_2, x_3 … ending up with a final guess of x_m.
   a. Issues here include both finding an initial guess x_1 and deciding on a stopping criterion that says that x_m, for some m, is good enough.