CS 205a Fall 2010 Midterm 1

Please write your name on the top right of the first page. The exam is closed book and no calculators are allowed. You have 1 hour and 15 minutes to complete the exam. There are two extra pages at the end if you run out of space. Please clearly state if you use those pages. **Total Points: 40.**

Quadrature (10 pts)

1. Given a function $f(x)$ and an interval $[a, b]$, state the Midpoint rule and the Trapezoidal rule that approximate $\int_a^b f(x)dx$. (6 pts)

2. What is the relationship between local error and global error? (2 pts) Prove it. (2 pts)

Interpolation (10 pts)

1. Construct the basis functions for Newton Interpolations of degree 2 at sample points $x = 0, 1, 2$. (3 pts)

2. Now constuct the basis functions for the Lagrange Interpolation of degree 2 at sample points $x = 0, 1, 2$. (3 pts)

3. Using the results from (2) and the same sample points, construct the Lagrange interpolation polynomial of degree 2 for the function $f(x) = x^2$. Show all the steps. (4 pts)

ODE/Finite Difference (10 pts)

1. Derive the order of accuracy of the central difference formula $f_c^{'}(x) = \frac{f(x+h)-f(x-h)}{2h}$, which is used to approximate the derivative of a function. (4 pts)

2. What is the difference between a hard and a stiff ODE. What kind of methods will you prefer for each and why? (3 pts)

3. Convert the second order ODE $mx''(t) = F$ to a system of first order ODEs. ($x''(t)$ denotes the second derivative of x with respect to time t.) (3 pts)

Conjugate Gradient (10 pts)

1. State the conjugate gradient algorithm. (4 pts)

2. For a search based method, we update our solution guess as $\vec{x}_{k+1} = \vec{x}_k + \alpha_k \vec{s}_k$, where $\vec{x}_k$ is the solution guess and $\vec{s}_k$ is the search direction at the k^{th} iteration. Let $\vec{e}_k = \vec{x}_k - \vec{x}_{exact}$ be the error the k^{th} iteration. Derive the value of α_k, such that $\vec{e}_{k+1}$ is orthogonal to $\vec{s}_k$ (i.e. all the remaining error is orthogonal to the current search direction). Your solution should be in terms of $\vec{e}_k$ and $\vec{s}_k$. (4 pts)

3. Now derive the value of α_k, such that $\vec{e}_{k+1}$ is A-orthogonal to $\vec{s}_k$. Also write your solution in terms of the residual $\vec{r}_k = -A\vec{e}_k$. (2 pts)