CS205 Homework #5

Problem 1

[Heath 5.5, p.248]

1. Show that the iterative method

$$x_{k+1} = \frac{x_{k-1}f(x_k) - x_k f(x_{k-1})}{f(x_k) - f(x_{k-1})}$$

   is mathematically equivalent to the secant method for solving a scalar nonlinear equation $f(x) = 0$.

2. When implemented in finite-precision floating-point arithmetic, what advantages or disadvantages does the formula given in part (1) have compared with the formula for the secant method (given in the notes and in Heath, section 5.5.4)?

Problem 2

[Heath 5.6, p.249] Suppose we wish to develop an iterative method to compute the square root of a given positive number y, i.e., to solve the nonlinear equation $f(x) = x^2 - y = 0$ given the value of y. Each of the functions g_1 and g_2 listed next gives a fixed-point problem that is equivalent to the equation $f(x) = 0$. For each of these functions, determine whether the corresponding fixed-point iteration scheme $x_{k+1} = g_i(x_k)$ is locally convergent to $\sqrt{y}$ if $y = 3$. Explain your reasoning in each case.

1. $g_1(x) = y + x - x^2$.

2. $g_2(x) = 1 + x - x^2/y$.

3. What is the fixed-point iteration *function* given by Newton's method for this particular problem?

Problem 3

[Heath 5.11, p.249] Suppose you are using the secant method to find a root x^* of a nonlinear equation $f(x) = 0$. Show that if at any iteration it happens to be the case that either $x_k = x^*$ or $x_{k-1} = x^*$ (but not both), then it will also be true that $x_{k+1} = x^*$.

Problem 4

[Heath 6.8, p.302] Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(\mathbf{x}) = \frac{1}{2}(x_1^2 - x_2)^2 + \frac{1}{2}(1 - x_1)^2$$

1. At what point does f attain a minimum?
2. Perform one iteration of Newton's method for minimizing f using as starting point $\mathbf{x}_0 = \begin{bmatrix} 2 & 2 \end{bmatrix}^T$
3. In what sense is this a good step?
4. In what sense is this a bad step?

[Note: The Newton method for optimization of a function of multiple variables $f : \mathbb{R}^n \to \mathbb{R}$ gives the update step $\mathbf{s}_k = \mathbf{x}_{k+1} - \mathbf{x}_k$ as the solution to the system $\mathbf{H}(\mathbf{x}_k)\mathbf{s}_k = -\nabla f(\mathbf{x}_k)$, where $\mathbf{H}(\mathbf{x}_k)$ is the Hessian matrix of f evaluated at $\mathbf{x}_k$. See also Heath, section 6.5.3.]