Investigation on Wave-guiding Properties of Nanotube Array^D

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Introduction

With the discovery of carbon nanotube (CNT), a lot of researches have been proposed on the properties of CNT and its possible applications. In this paper, the wave-guiding properties of single-wall carbon nanotube (SWCNT) arrays are discussed to explore the possibility of applying SWCNT arrays as the microwave transmission line. According to the qualitative analysis [1], periodic conducting rods have band pass properties if the height of the rod h satisfies the condition of $0.3\lambda_0 < h < 0.5\lambda_0$, where λ_0 is the free-space wavelength of an electromagnetic wave. As SWCNT arrays' geometries are similar to those of macroscopic metallic rod arrays, similar band-pass properties should exist. Meanwhile, the wavelength λ on the SWCNT is far shorter than λ_0 in the free space [2-4], due to the quantum effect. Thus, the SWCNT-built transmission line should have a much smaller dimension compared to traditional metallic rods working at the same frequency. In this paper, first, the semi-classical model of SWCNT [4] has been used to deduce the Hallén's type integral equation. Then rigorous numerical method is employed to analyze a one-row SWCNT array. Finally, the similarity and differences between SWCNT array and traditional periodic conducting rods are also discussed.

Formulation

Consider a one-row SWCNT array shown in Fig.1.



Fig.1 aligned cylindrical tube array

The impedance of a SWCNT is given by [4]

$$Z_{impedance} = \frac{1}{2\pi a\sigma_{cn}} = \frac{\pi\hbar\upsilon}{4e^2v_F} + j\omega\frac{\pi\hbar}{4e^2v_F}$$
(1)

Where *a* is the radius of SWCNT, v_F is the Fermi velocity, ω is the operating angular frequency, and v is the relaxation frequency. According to [4-6], the axial component of the electric field of an incident wave $E_z^{i(n)}$ and the current $I^{(n)}$ on the *n*th SWCNT should satisfy Hallén's type integral equation.

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$$\sum_{i \neq n-L} \int_{-L}^{L} I^{(i)}(z') K^{(in)}(z-z') dz' + \int_{-L}^{L} I^{(n)}(z') (K^{(nn)}(z-z') + q(z-z')) dz'$$

$$= c_{n1} \cos kz + c_{n2} \sin k |z| - j \frac{4\pi\omega\varepsilon}{k} \int_{0}^{z} E_{z}^{i(n)}(s) \sin k(z-s) ds$$
(2)

where

$$K^{(nn)}(z-z') = \frac{\exp(-jk\sqrt{(z-z')^2 + a^2})}{\sqrt{(z-z')^2 + a^2}}, K^{(in)}(z-z') = \frac{\exp(-jk\sqrt{(z-z')^2 + (d \times |i-n|)^2})}{\sqrt{(z-z')^2 + (d \times |i-n|)^2}}, \quad i \neq n$$

$$q(z-z') = \frac{2\pi\omega\varepsilon\exp(-jk|z-z'|)}{Z_{impedance}^{-1} \cdot k}$$

By solving equation (2) with MoM programs, the current distribution on each SWCNT can be obtained. Further, the passband of the transmission line as well as the electromagnetic field distribution along the transmission line can be calculated. It should be noticed that the integral equation (2) can be applied to both macroscopic metallic and SWCNT arrays, the only difference between them lies in the different impedance $Z_{impedance}$ between macroscopic metallic rods and SWCNTs.

Wave-guiding performance of the SWCNT array

Fig.2 (a) shows the typical pass band of a macroscopic metallic rod array, and Fig.2 (b) is the guiding energy along the array. The standing wave pattern along the array is caused by the unmatched load end.



Fig2(a) passband of macroscopic metallic rod array, tube length=1cm, diameter=1mm, distance=3mm, N=50, where N is the number of conducting cylinders; the y axis indicates the maximum current on the last rod

Fig2(b) absolute value of z component of magnetic vector along the rod array in the z=0 plane at the frequency of 11GHz; the values are simulated at a distance of 1mm from the array

Different from the macroscopic metallic rods, the wavelength on the SWCNT is much shorter than that in the free space [4]. Therefore, if SWCNT array and macroscopic metallic rods have similar wave-guiding properties, the passband requirement $0.3\lambda_0 < h < 0.5\lambda_0$ [1] should be changed to $0.3\lambda < h < 0.5\lambda$, where λ is the wavelength on SWCNT. Thus, the transmission lines constructed by SWCNT array will be much smaller than traditional metallic rods working at the same frequency. This analysis is confirmed by the results shown in Fig.3, in which the pass band and λ_0 are around 500GHz and 0.6mm, respectively. To ensure the same pass band requirement, the length of macroscopic metallic rods should be about 0.18mm at least. As comparison, the SWCNT studied in Fig.3 has a length of only 15 μ m, indicating that the wavelength on the SWCNT array is only about 1/12 of the wavelength in the free space. Furthermore, Fig.3 (b) shows that energy is guided effectively along the array in the passband. The decay near the input port can be regarded as the transition between different modes and the standing wave pattern of energy distribution along the array, as mentioned before, is caused by the reflected wave from the unmatched load end.



Fig.3(a) passband of SWCNT array, tube length=15 μ m, diameter=5.424nm, distance=100nm, τ =3ps, N=100. The y-axis indicates the maximum current amplitude on the SWCNT in the end

Fig.3(b) absolute of z component of magnetic vector along the SWCNT array in the z=0 plane at the frequency of 483GHz

The guide wavelength on SWCNTs

Based on the transmission line model [3] of the single SWCNT dipole, the guide wavelength on SWCNT can be roughly predicted to be $0.01\lambda_0$, where the kinetic inductance plays the primary role in shrinking the wavelength. According to quantitative calculation [4] of the SWCNT dipole, the wave-guiding wavelength on SWCNT is about $0.02\lambda_0$. Different from the structures in [3] and [4], the present transmission line consists of SWCNT array and the mutual coupling between SWCNTs makes the situation much more complex. In order to identify the effect of mutual coupling on the current distribution on the SWCNTs, a situation of two SWCNTs placed in parallel under the excitations of common and differential mode is analyzed at the resonance frequency of a single SWCNT dipole. Fig.4 shows the current distributions on two SWCNTs under excitation of common mode and differential mode at the frequency of 210GHz. As comparison, the current distribution on one single SWCNT at the same frequency is also given. The variations of current distributions in Fig.4 reveal the changing of resonance frequency in different cases. It can be found that the wavelength on SWCNTs gets longer under common mode excitation while the wavelength gets shorter under differential mode excitation. This can be qualitatively explained briefly. When two coupling SWCNTs are placed in parallel and excited by common mode, the currents on the two SWCNTs are in the same phase. In this case, the equivalent inductance per unit length of the structure is smaller, which leads to the increasing of the phase velocity and thus the increasing of wavelength on SWCNTs. On the other hand, excitation of the two coupling SWCNTs with differential mode leads

to the decreasing of the phase velocity and thus the decreasing of wavelength on SWCNTs. Generally, when SWCNT array is used as a transmission line, the phases and magnitudes of incident waves on neighboring SWCNTs are nearly the same, which is similar to the situation under common excitation. Thus the guide wavelength on SWCNTs is much larger than the plasmon wavelength on single SWCNT, which is approximately 0.02 of the wavelength in the free space. [4]





Fig.4(a) current distribution on the single SWCNT at the frequency of 210 GHz

Fig.4(b) current distribution on SWCNT under excitation of common mode and differential mode; 210GHz

Conclusion

A novel type of transmission line constructed by SWCNT array is proposed and analyzed with MoM. When the SWCNT array is properly aligned, the electromagnetic wave can be guided along the array effectively. Moreover, the dimensions of transmission line can greatly shrink, due to quantum effect. Finally, the mutual coupling between SWCNTs is analyzed, which indicates the case of SWCNT array is more complex than the case of single SWCNT dipole.

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