# Effects of Earth's Curvature in Full-wave Modeling of VLF Propagation

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## Introduction

Effects of curvature are included in full-wave finite-element approach to calculate wave propagation in horizontally stratified waveguides. The first stage of the research focuses on formulating and verifying the approaches. It is shown that the inclusion of curvature improves accurary in calculate wave propagation in curvilinear stratified systems.

## Maxwell Equations

For convenience we use SI units for electric field and use modified units for H, so that $H=Z_0 H_{SI}$ with $Z_0$ as the wave impedance in the free space [*Lehtinen and Inan, 2009*]. Maxwell equations can be expressed as:

$$\left(\hat{U}_X \frac{D_X}{ik_0}+\hat{U}_Y \frac{D_Y}{ik_0}+\hat{U}_Z \frac{D_Z}{ik_0}\right) F=\hat{K}\, F$$

Where $D_X$ , $D_Y$ , $D_Z$ are partial derivatives in the three axes of a general curvilinear coordinate; $\hat{U}_X$ , $\hat{U}_Y$ , $\hat{U}_Z$ are matrices from *Altman and Suchy [1991, p. 21].* $\hat{K}$ and $F$ are defined as:

$$\begin{pmatrix} \hat{\epsilon}_k & 0 \\ 0 & \hat{\mu}_k \end{pmatrix}=\hat{K}\,, \quad \begin{pmatrix} E \\ H \end{pmatrix}=F$$

In planar othorgonal systems, $D_X$ , $D_Y$ , $D_Z$ degenerates to $\partial_X,\ \partial_Y,\ \partial_Z$ .

## General Othogonal Curvilinear Stratified System

The new approach generalizes the conservation of wave number components from plane-stratified systems to general curvilinear stratified systems.

Let us consider a general orthogonal curvilinear coordinate system (X, Y, Z) with translational symmetry along (X, Y), in which system the metric is given by:

$$ds_X=h_X\, dX=(1+Z\alpha_X)\, dX$$
$$ds_Y=h_Y\, dY=(1+Z\alpha_Y)\, dY$$
$$ds_Z=h_Z\, dZ=(1+Z\alpha_Z)\, dZ$$

Where $\alpha_X, \alpha_Y, \alpha_Z$ play roles of curvatures in corresponding directions. For example, for cylindrical system with Y as the axial direction and Z as the radial direction, the rescaling factors in X and Y directions are given as:

$$\alpha_X=1/R,\quad \alpha_Y=0$$

where R is the radius of the earth; $\alpha_Z$ describes distance rescaling along Z axis. In the curvlinear stratified system we discuss, $\alpha_Z$ can be always set to zero, thus making Z simply mean the the altitude from the earth surface.

When $\alpha_X, \alpha_Y, \alpha_Z$ all equals zero, the curvi-linear system degenerates to a planar system.

## Clemmow-Heading Operator

Finding $n_Z$ , the effective refractive index in z direction, is one of the key steps in full-wave modeling. One method is to find $n_Z$ as the eigenvalues of the operator L which translates electromagnetic fields forward in z-direction *[Clemmow and Heading, 1954*]. Our approach includes the curvature effect into the operator L by adding the recaling factors in each axis. The eigenvalues of the modified operator L including the curvature effect are the effective refractive indices in z direction for the curvilinear stratified system.

$$\frac{\partial_Z}{ik_0} F_p=LF_p$$

$$L=B\left(A_{pZ} K_{ZZ}^{-1} A_{Zp}-K_{pp}+C\right)$$

$$A=\frac{\beta_X}{h_X}\hat{U}_X+\frac{\beta_Y}{h_Y}\hat{U}_Y-\hat{K}\,,\quad B=-\hat{U}_{Z,pp}\,,$$

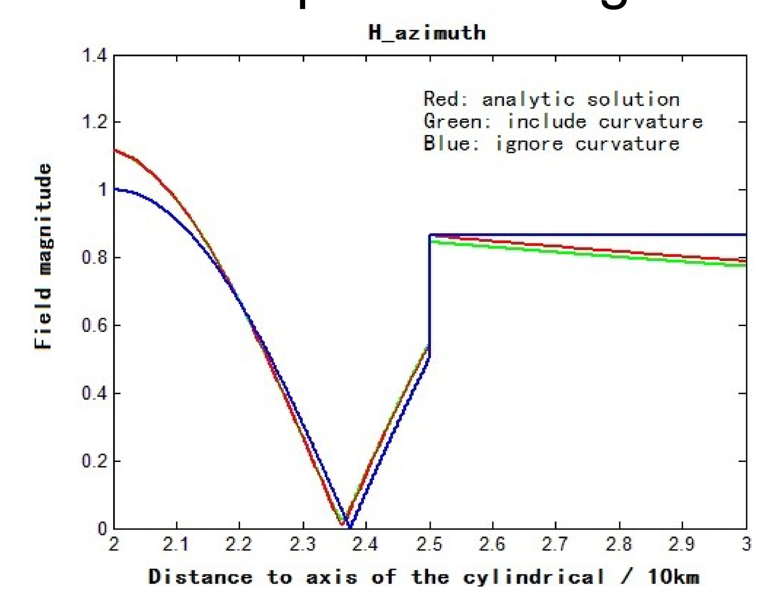
$$C=\frac{\alpha_X}{ik_0 h_X} B_1+\frac{\alpha_Y}{ik_0 h_Y} B_2,$$

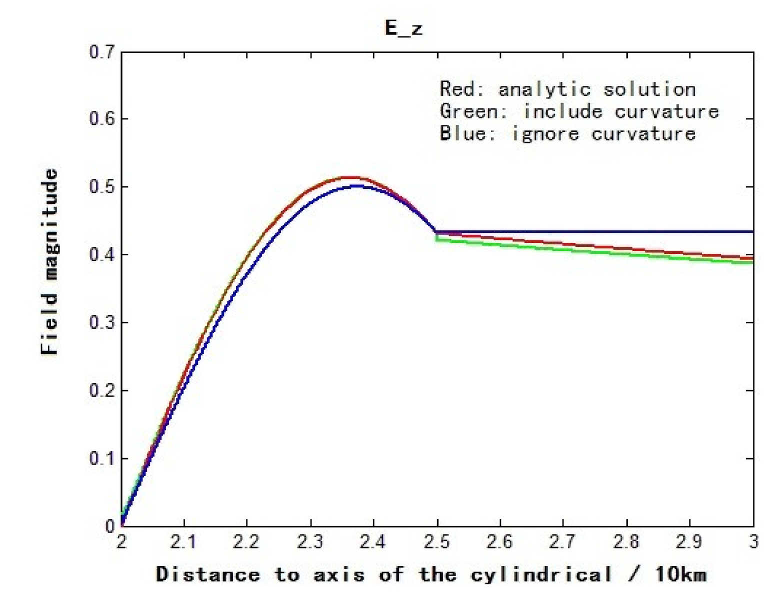
$$B_1=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},\ B_2=\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In the formulation above, $\beta_X$ and $\beta_Y$ are the conserved refractive index components in X and Y directions in the curvilinear stratified system. Note that the subscript p indicates the columns and rows that correspond to X and Y directions.

## Validation of Codes

The codes written based on the curvature-included full-wave approach are applied to calculate the wave propagation in a cylindrical system with uniform isotropic medium. The results of numerical simulation are compared with analytic solutions, showing improved accuracy compared with codes of planar system full-wave approach. The validated codes can readily be applied to non-uniform and anisotropic medium with arbitrary curvatures in X and Y directions, such as the earth-ionosphere waveguides.




## Future work

This work formulates and validates the full-wave approach including curvature effects, showing improved accuracy compared with the full-wave approach ignoring curvature. The approach proposed in this work will be applied to calculate wave propagation in the earth-ionosphere waveguide and compared with ground and satellite observations.

## Reference

Altman, C., and K. Suchy (1991), Reciprocity, Spatial Mapping and Time Reversal in Electromagnetics, Kluwer Academic Publishers, Boston.

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