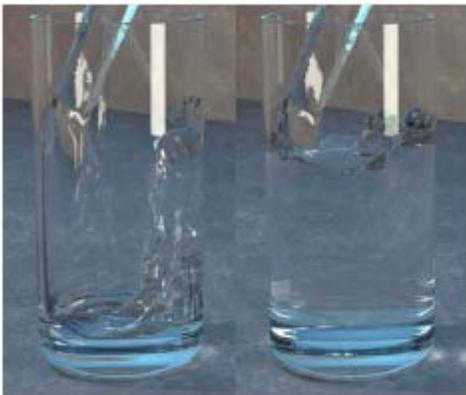


## Making a Computational Splash

Ronald Fedkiw, Stanford University

Have you ever sat in a movie theater with a box of popcorn and a soft drink watching a movie like the “Perfect Storm” and wondered where Hollywood found a cameraman brave enough to operate a camera under such dangerous conditions? Well, that particular cameraman was sitting in a small office north of San Francisco at a company called Industrial Light and Magic, which was founded in 1975 by George Lucas of Star Wars fame. There, a computer graphics specialist, our “cameraman”, safely operated the camera with the mouse attached to his computer. In fact, the camera was little more than a computer program that calculated the relative positions of boats and waves in order to add captivating imagery to a tale about fishermen struggling with treacherous waves on rough seas. Hollywood films are created to captivate and entertain, and thus they frequently contain exciting scenes with natural phenomena such as water waves, smoke, tornados or even lava erupting from volcanoes. Obviously, we’re not going to put George Clooney on a boat in the middle of the Atlantic Ocean and subsequently pulverize it with waves until it sinks. In fact, we’re not going to put anyone on a boat in such a treacherous storm, assuming we could even find such a storm in the first place. It turns out to be a lot easier to make waves out of math than out of water.



**Figure 1: Computer generated water being poured into a glass.**

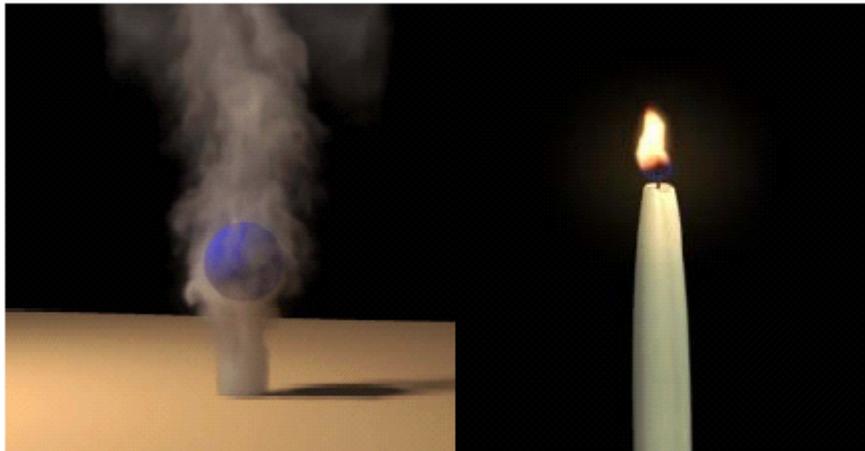


**Figure 2: A ball making a splash in a tank of computer generated water.**

For centuries, applied mathematicians and physicists have derived mathematical equations describing the behavior of a variety of substances including water waves in the ocean and the metal hull of a ship. These equations are quite complicated and can only be solved in special situations or with the aid of simplifying assumptions that usually rule out problems of practical interest. However, in the last half century, the advent of computer technology has led to

a revolution in the study of these types of equations. Using approximation theory, numerical analysts have devised a number of algorithms that enable one to program computers to estimate solutions to many of the equations governing the physical world, to any desired numerical accuracy. Moreover, these numerical solutions provide useful information for practical problems of interest to both scientists and engineers.

Solving such problems falls into an area of research is referred to as “scientific computing.” Scientific computing has become the third branch of research in many engineering departments joining theory and experiment as classical approaches to obtaining information about the world around us. While scientific computing has classically been applied to physical problems such as those faced in mechanical, aerospace, and structural engineering, the fastest growing application areas may currently be in electrical and computer engineering, biology, and medicine. More broadly, scientific computing encompasses both computational science and computational mathematics. We emphasize that mathematics is the fundamental language for problem solving, and when confronted with a problem most scientific researchers attempt to formulate a mathematical description of that problem. Once the mathematical description exists, the problem is potentially solvable on a computer using either existing algorithmic techniques or newly devised methods. Thus, scientific computing has become a fundamental requirement for solving problems in signal processing, image analysis, robotics, computer vision, human computer interaction and computer graphics.



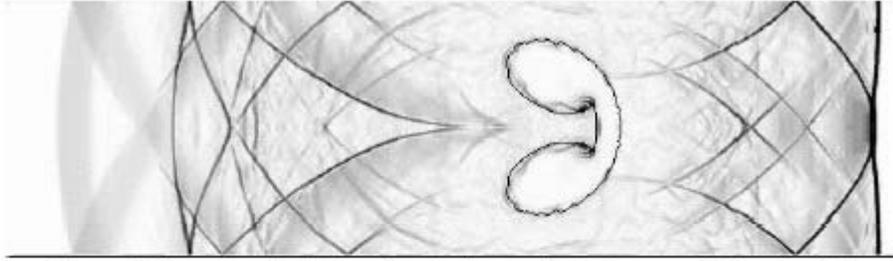
**Figure 3: Computer generated smoke rolls past a sphere (left), and a computer generated candle flame (right).**

Returning to our cameraman and his goal of captivating us with dramatic sequences of waves pounding on a small ship, we now see that there are a lot of potential resources for creating “special effects” waves on the computer. In the particular case of the “Perfect Storm”, Industrial Light and Magic (ILM) had recently hired a professor of atmospheric science (who specializes in

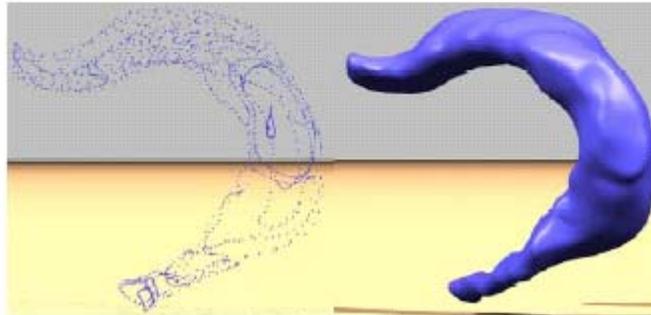
scientific computing) to use his knowledge of oceans and waves, especially his algorithmic knowledge, to construct numerical solutions of large crashing waves on the ILM computers.

The “Perfect Storm” movie was not the first instance of scientific computing algorithms being used to make movies of boats and waves. A few years earlier, a Department of Defense (DoD) oriented company called Arete Associates started a small spin-off company called Arete Entertainment to create computer graphics software that constructs artificial, but surprisingly realistic, ocean wave models. One can imagine that it could be particularly useful to be able to reproduce ocean wave models based on, for example, current wind speeds. If one could measure the wind and estimate what the ocean wave pattern should look like, this could be compared to the actual wave patterns in order to ascertain if there has been any recent outside influences on the wave patterns. That is, one could subtract the estimated wave pattern from the current wave pattern revealing the wake of a large ship that passed through the water even some time ago. This becomes a lot more interesting when one realizes that underwater ships (submarines) can create surface disturbances as well. The implications to the surveillance community are obvious. Using their knowledge of scientific computing algorithms for studying waves, Arete Entertainment developed computer graphics software to create ocean waves based on wind speeds. This software has won numerous awards including a Technical Academy Award, and was used to create ocean waves in many films such as “Water World” and James Cameron’s “Titanic”. Another recent film, “Cast Away”, used similar technology to simulate a plane crashing into the ocean stranding Tom Hanks on a deserted island.

As mentioned earlier, scientific computing algorithms can be used to simulate a variety of natural phenomena including the water shown in figures 1 and 2, and the smoke and fire shown in figure 3. While these every day events (smoke, fire and water) are easy to relate to, similar numerical techniques can also be applied to events that the human eye will miss completely. For example, figure 4 shows a computer simulation of a high-speed shock wave interacting with a Helium bubble. Scientific computing is having a growing impact in the fields of imaging and data analysis as well. This is important, for example, in medicine, biology and even surveillance. For example, figure 5 shows several data points obtained from MRI imaging of a rat’s brain along with a three-dimensional geometric reconstruction of the rat brain obtained using numerical methods on a computer.

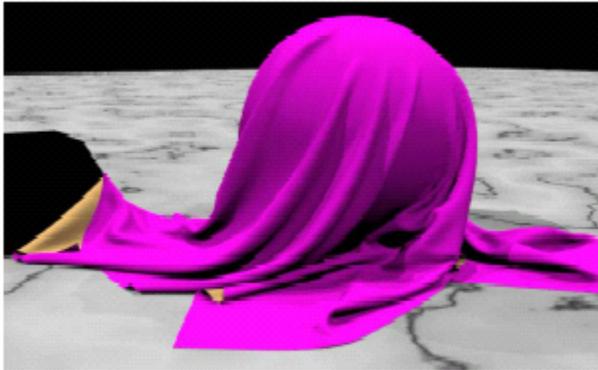


**Figure 4: Computer simulation of a shock wave impinging on a Helium bubble.**



**Figure 5: MRI data points from a rat's brain along with a computer reconstruction of the brain geometry.**

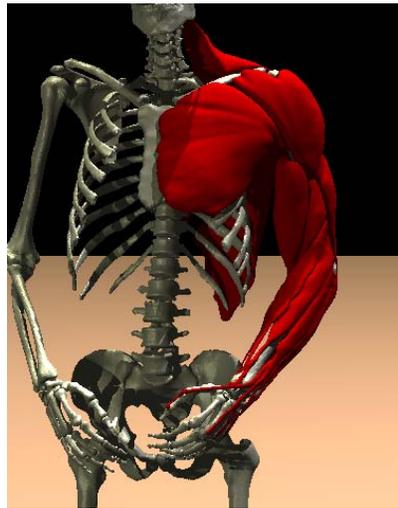
Possibly the most exciting area for future applications of scientific computing is the computer simulation and study of humans themselves. Researchers in biomechanics and medicine are currently working to write down mathematical equations and numerical models that describe most of the human body from the cardiovascular system to muscles, bones and organs. As these equations and models are formulated, there is an ever-growing need for new computational algorithms that can be used for the computer simulation of biological structures. Figure 7 shows an example computer simulation of a skeleton running. Geometric models were used for the bones, mathematical equations were used to describe the limited motion allowed for by the connective tissue in joints, and special models for soft tissue were used to simulate the muscles (the red regions in figure 7 represent the biceps muscles). Figure 8 shows 30 muscles of the upper limb represented as a tetrahedral mesh ready for finite element simulation. Computer simulation of humans is of interest to a wide variety of commercial industries as well. The entertainment industry would like to simulate virtual actors, the textile industry would like to simulate both runway models and everyday customers trying on virtual clothing, etc. See figure 6 for a computer simulation of a piece of draped cloth.



**Figure 6: Computer simulation of a piece of draped cloth.**



**Figure 7: Computer simulation of a running skeleton with biceps.**



**Figure 8: A tetrahedral muscle mesh ready for finite element simulation.**

As both everyday personal computers and national laboratory supercomputers continue to increase in speed and as better algorithms are developed, the size, complexity and realism of the problems that can be simulated on these computers increases as well. In addition, as researchers branch out into new and exciting research areas, they will formulate mathematical descriptions of their problems that are subsequently amenable to computer simulation. The future is bright in a number of research areas, and where researchers go, math and computer algorithms are sure to follow.

### **Sidebar**

The mathematical equations that describe the movement of liquids and gases are known as the Navier-Stokes equations. These equations describe how fluid particles move through space, the effects of internal friction or viscosity, and the way sound waves are transmitted through a

fluid. While viscosity is relatively straightforward to account for, the particle motion and sound wave transmission can be rather difficult to deal with. Computational methods for approximating these effects have to determine which way the particles and the sound waves are moving (and they usually move in different directions) and account for both this directionality and the speed of propagation. When modeling high-speed gas flows containing shock waves as shown in figure 4, it is important to accurately resolve both the particle motion and sound wave transmission effects. On the other hand, when modeling sloshing water or other liquids as shown in figures 1 and 2, precise treatment of the sound waves is not necessary in order to obtain adequate solutions of the equations. Moreover, the treatment of fast moving sound waves would make the problem computationally much more expensive to solve. To remedy this difficulty, liquids can be assumed to be incompressible, that is they preserve volume. This assumption removes the directional component and the stiffness of the sound waves making the problem computationally tractable. Moreover, this is a physically realistic assumption for liquids, especially when one considers that a liquids' resistance to compression is responsible for the strong forces produced by hydraulics devices.